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TRIPLE CONFIGURATIONS OF TRAVELING SHOCK WAVES IN INVISCID GAS FLOWS

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Triple configurations of shock waves in supersonic inviscid flows of a perfect gas are considered. The basic parameters of triple configurations are determined, and the conditions of solution existence are analyzed.

Key words: *triple configuration, shock wave, von Neumann configuration.*

1. Initial Relations. Local processes inherent in interference of traveling shock waves are studied. A triple configuration (TC) of shock waves with one tangential discontinuity, which have a common triple point, is considered as a shock-wave system. The gas flow is divided into two flows, one of them passing through the first and second shock waves, and the other passing through the third shock wave. After passing through the shock waves, the flows do not mix with each other: they are separated by the tangential discontinuity τ moving in space. Depending on the orientation of shock waves with respect to the incoming flow, triple configurations are classified into three types: TC-1, TC-2, and TC-3. The triple configuration of each type is an important particular case in problems of interaction of opposing shock waves, of a shock wave with a plane target (Mach reflection), and of up-catching shock waves (Fig. 1).

The model of triple configurations proposed ignores the gas viscosity and heat conduction outside the shock-wave fronts. The shock-wave fronts and also the front of the tangential discontinuity are assumed to be plane on the interference line of interaction. This model does not describe phenomena that occur in transonic flows, but it seems reasonable to use this model at Mach numbers $\mathrm{M}_0 > 1.5$ if the original undisturbed flow is homogeneous. The thermodynamic model is based on a perfect gas whose state is described by the equations

$$p = RT/v, \qquad c_v = \text{const}.$$

The local problem of interference can be always reduced to a plane problem (it is possible to draw a plane through a point on the triple line, which is perpendicular to the triple line in the neighborhood of this point). We use the coordinate system Oxy with the x axis directed along the free-stream velocity vector $\boldsymbol{V}_0$. In the general case, the triple point moves with respect to the coordinate system with a velocity W. The two quantities determined by the coordinate system are the velocities D_1 and D_3 of the first and third shock waves (for $D_1 > 0$ and $D_3 > 0$, the projections of these velocities onto $\boldsymbol{V}_0$ are greater than zero).

All parameters of the initial gas flow (namely, the pressure p_0, the specific volume v_0, the temperature T_0, and the velocity $\boldsymbol{V}_0$) are assumed to be known. In addition, the pressure p_1 behind the front of the first shock wave is assumed to be known. The free-stream parameters are indicated by the zero subscript, and the flow parameters behind the shock waves are indicated by the subscripts corresponding to the shock-wave number.

In what follows, we use dimensionless parameters. The shock-wave intensity is determined as the ratio of the static pressure of the gas behind the wave to the static pressure ahead of the wave:

$$J_i = p_i/p_j.$$

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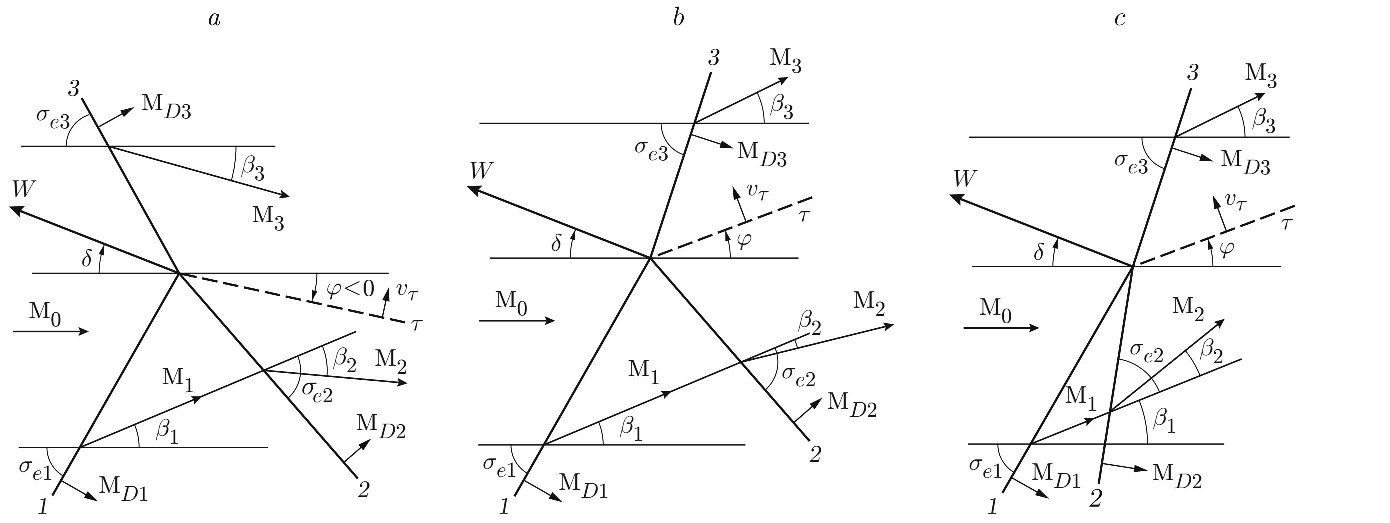

Fig. 1. Types of triple configurations of the first shock wave (1), second shock wave (2), and third shock wave (3) for TC-1 (a), TC-2 (b), and TC-3 (c).

Here

$$i = 1, 2, 3, \qquad j = \begin{cases} 0, & i = 1, 3, \\ 1, & i = 2. \end{cases}$$

Instead of gas velocities, we consider the Mach numbers $\mathrm{M}_0 = V_0/a_0$ and $\mathrm{M}_i = V_i/a_i$ (a_0 and a_i are the local velocities of sound in the gas in the corresponding regions). Instead of the shock-wave velocities, we use the Mach numbers (Fig. 1)

$$\mathrm{M}_{Di} = D_i/a_j.$$

The triple configuration is defined by four parameters: free-stream Mach number M_0, intensity of the first shock wave J_1, and Mach numbers of the first and third shock waves M_{D1} and M_{D3} (Fig. 1). The sought parameters are the intensities of the second (J_2) and third (J_3) shock waves. All the remaining gas-dynamic parameters [Mach numbers behind the shock-wave fronts M_i, angles of inclination of these waves σ_{ei}, angles of flow turning on the shock-wave fronts β_i, Mach number of the second shock wave M_{D2}, angle φ between the tangential discontinuity and the x axis, which is counted in the direction of the second shock wave, and angle δ characterizing the direction of motion of the triple point (Fig. 1)] are determined via the sought quantities. We consider the angles σ_{ei} in the interval $[0; \pi/2]$. The first shock wave always arrives at the point of wave interference; the signs of the quantities ψ_2 and ψ_3 characterize the second and third shock waves, respectively [$+1$ if the angle σ_{ei} faces the direction of the triple point and -1 in the opposite direction (Fig. 1)]. Therefore, we have $\psi_2 = \psi_3 = -1$ for TC-1, $\psi_2 = -1$ and $\psi_3 = +1$ for TC-2, and $\psi_2 = \psi_3 = +1$ for TC-3. We also consider the configuration TC-4 for which $\psi_2 = +1$ and $\psi_3 = -1$.

In the steady-state case, the problem formulated is reduced to a six-order equation with respect to the intensity of the second shock wave J_2 for the same defined parameters [1]. The remaining flow parameters in [1] are presented in explicit form via the defined parameters and J_2. The study of the unsteady problem was started in [2].

We use the conditions of equality of static pressures on both sides of the tangential discontinuity τ and gas-velocity components normal to the surface of the tangential discontinuity:

$$J_1 J_2 = J_3; \tag{1}$$

$$\mathrm{M}_2 a_2 \sin(\beta_1 + \psi_2\beta_2 - \varphi) = v_\tau = \mathrm{M}_3 a_3 \sin(\psi_3\beta_3 - \varphi). \tag{2}$$

In steady-state triple configurations, the gas-velocity components normal to the surface of the tangential discontinuity are equal to zero. The angles of inclination of the shock waves σ_{ei}, the angles of flow turning β_i, and the

Mach numbers behind the shock waves M_i are functions of the shock-wave intensity, its Mach number, and the flow Mach number ahead of the shock wave only. These functions have the form [3]

$$\sigma_{ei} = \arcsin \frac{M_{Di} - \chi_i I_i}{M_j}, \qquad I_i = \sqrt{\frac{J_i + \varepsilon}{1 + \varepsilon}}, \qquad \varepsilon = \frac{\gamma - 1}{\gamma + 1}; \tag{3}$$

$$\begin{gathered} \tan \beta_i = -\frac{\cos \sigma_{ei}}{\sin \sigma_{ei} + \chi_i M_j/((1-\varepsilon)(I_i - 1/I_i))}, \\ \chi_i = -1\colon \quad \beta_i \in [0, \pi], \qquad \chi_i = +1\colon \quad \beta_i \in [-\pi/2, \pi/2]; \end{gathered} \tag{4}$$

$$\frac{J_i(1 + \varepsilon J_i)}{J_i + \varepsilon} M_i^2 = M_j^2 + 2\chi_i(1-\varepsilon) \frac{I_i^2 - 1}{I_i} M_j \sin \sigma_{ei} + (1-\varepsilon)^2 \frac{(I_i^2 - 1)^2}{I_i^2}, \tag{5}$$

where γ is Poisson's ratio; the parameter χ_i has the value $+1$ if the projection of the gas velocity vector onto the normal to the shock wave is smaller than the shock-wave velocity and $\chi_i = -1$ if this projection is greater than the shock-wave velocity.

The conditions of intersection of the surfaces of the shock waves and the surface of the tangential discontinuity over one line (see Fig. 1) are written as

$$\begin{gathered} D_1 = -W \sin(\delta + \sigma_{e1}), \qquad D_2 = -W \sin(\psi_2 \delta + \psi_2 \beta_1 + \sigma_{e2}), \\ D_3 = -W \sin(\sigma_{e3} + \psi_3 \delta), \qquad v_\tau = W \sin(\delta + \varphi), \end{gathered} \tag{6}$$

where v_τ is the normal component of velocity of the tangential discontinuity.

2. Calculation of the Basic Parameters of the Triple Configurations. The quantities M_1, σ_{e1}, and β_1 are determined from Eqs. (3)–(5). Let us express the quantities M_2, M_3, σ_{e2}, σ_{e3}, β_2, β_3, M_{D2}, J_3, φ, and δ as functions of J_2. We find J_3 from Eq. (1), σ_{e3} from Eq. (3), and δ from the first and third formulas of Eqs. (6):

$$\delta = \arctan \frac{M_{D3} \sin \sigma_{e1} - M_{D1} \sin \sigma_{e3}}{\psi_3 M_{D1} \cos \sigma_{e3} - M_{D3} \cos \sigma_{e1}}. \tag{7}$$

Substituting Eq. (3) into the second formula of Eqs. (6), we obtain the expression for the Mach number of the second shock wave

$$M_{D2}(M_0, J_1, J_2, M_{D1}, M_{D3}) = \frac{-\chi_2 I_2 (z \cos(\delta + \beta_1) - 1) \pm \sqrt{\hat{D}}}{z^2 - 2z \cos(\delta + \beta_1) + 1}, \tag{8}$$

where

$$\hat{D} = [M_1^2 (z^2 - 2z \cos(\delta + \beta_1) + 1) - I_2^2 z^2] \sin^2(\delta + \beta_1); \tag{9}$$

$$z = \sqrt{\frac{J_1(1 + \varepsilon J_1)}{J_1 + \varepsilon}} \, \frac{M_1 \sin(\psi_3 \sigma_{e1} - \sigma_{e3}) \cos \delta}{\psi_3 M_{D1} \cos \sigma_{e3} - M_{D3} \cos \sigma_{e1}}. \tag{10}$$

Equation (8) holds if the following conditions are satisfied:

$$\hat{D} \geq 0, \qquad \psi_2 \cos \sigma_{e2} \sin(\beta_1 + \delta)(M_{D2} z / M_1 - \sin \sigma_{e2} \cos(\beta_1 + \delta)) \geq 0.$$

Then, σ_{e2} is determined from Eq. (3). Relations (4) and (5) determine M_2, M_3, β_2, and β_3. In accordance with Eq. (2), φ is written as

$$\varphi = \arctan \frac{x \sin(\beta_1 + \psi_2 \beta_2) - y \sin \psi_3 \beta_3}{x \cos(\beta_1 + \psi_2 \beta_2) - y \cos \psi_3 \beta_3},$$

where

$$x = M_2 \sqrt{\frac{(1 + \varepsilon J_1)(1 + \varepsilon J_2)}{(J_1 + \varepsilon)(J_2 + \varepsilon)}}, \qquad y = M_3 \sqrt{\frac{1 + \varepsilon J_1 J_2}{J_1 J_2 + \varepsilon}}.$$

Substituting Eq. (2) into the fourth formula of Eqs. (6), we obtain the equation for the unknown $J_2(M_0, J_1, M_{D1}, M_{D3})$:

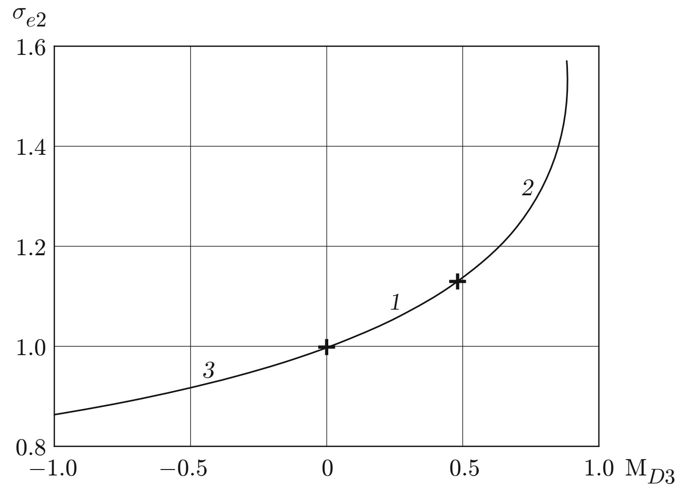

Fig. 2. Angle of inclination of the second shock wave versus the Mach number of the third shock wave for TC-1 and TC-2 with $J_1 = 2$: segments 1 and 2 refer to the solutions 1 of the types TC-1 and TC-2, respectively; segment 3 refers to the solution 2 of the type TC-1; the points refer to the transitions from one type of the solution to another.

$$1 + \frac{\mathrm{M}_3}{\mathrm{M}_1} z \sqrt{\frac{J_2(1+\varepsilon J_1 J_2)(J_1+\varepsilon)}{(J_1 J_2+\varepsilon)(1+\varepsilon J_1)}} \, \frac{\sin(\psi_3\beta_3-\varphi)}{\sin(\delta+\varphi)} = 0. \tag{11}$$

The gas-dynamic parameters are calculated in the frame of reference fitted to the first shock wave ($\mathrm{M}_{D1} = 0$), i.e., the interference of the quiescent and traveling shock waves is studied. We consider the flow of a diatomic gas ($\varepsilon = 1/6$) with a free-stream Mach number $\mathrm{M}_0 = 3$. The gas passes through the first shock wave from left to right; therefore, we have $\chi_1 = -1$. In addition, the gas passes through the second shock wave from left to right; hence, we have $\chi_2 = -1$ (Fig. 1). Let us consider the case where the gas passes through the third shock wave from left to right as well, i.e., $\chi_3 = -1$. For $\mathrm{M}_{D1} = 0$, Eq. (7) yields $\delta = -\sigma_{e1}$, i.e., δ is independent of M_{D3}. The plus and minus signs in Eq. (8) correspond to the solutions 1 and 2, respectively.

Equation (8) yields two relations for the Mach number of the second shock wave; solutions of the types TC-1, TC-2, TC-3, and TC-4 can exist for each relation. The laws of conservation of mass, momentum, and energy and the law of increasing entropy are valid in all cases. The solutions cannot be chosen within the framework of the local problem. All cases were considered in the calculations performed.

The solution is analyzed for the quantities J_2 and σ_{e2}. The intensity of the second shock wave J_2 is chosen as the basic thermodynamic parameter, and the angle of inclination of the second shock wave σ_{e2} is used as the basic geometric parameter. The dependence $\sigma_{e2}(\mathrm{M}_{D3})$ for TC-1 and TC-2 with $J_1 = 2$ is plotted in Fig. 2.

In the steady-state case ($\mathrm{M}_{D1} = \mathrm{M}_{D3} = 0$), the solutions corresponding to different signs in Eq. (8) coincide. To study the continuity of the solution obtained in the neighborhood of the point $\mathrm{M}_{D3} = 0$, we use relations (8)–(10), which transform to the following relation with allowance for the equality $\mathrm{M}_{D1} = 0$:

$$z = -\sqrt{\frac{J_1(1+\varepsilon J_1)}{J_1+\varepsilon}} \, \mathrm{M}_1 \sin(\psi_3\sigma_{e1}-\sigma_{e3}) \, \frac{1}{\mathrm{M}_{D3}}, \qquad \hat{D} = (\mathrm{M}_1^2 - I_2^2) z^2 \sin^2(\delta+\beta_1) + O\Big(\frac{1}{\mathrm{M}_{D3}}\Big),$$

$$\mathrm{M}_{D2} = \frac{I_2 \cos(\delta+\beta_1) \pm \sqrt{\mathrm{M}_1^2 - I_2^2} \, |\sin(\delta+\beta_1)| \, \mathrm{sign}\,(z)}{z} + O(\mathrm{M}_{D3}^2).$$

From here, we obtain

$$\mathrm{M}_{D2} = \frac{-I_2 \cos(\delta+\beta_1) \pm N \mathrm{sign}\,(\mathrm{M}_{D3})}{\sqrt{J_1(1+\varepsilon J_1)/(J_1+\varepsilon)} \, \mathrm{M}_1 \sin(\psi_3\sigma_{e1}-\sigma_{e3})} \, \mathrm{M}_{D3} + O(\mathrm{M}_{D3}^2),$$

$$N = \sqrt{\mathrm{M}_1^2 - I_2^2} \, |\sin(\delta+\beta_1)| \, \mathrm{sign}\,(\psi_3\sigma_{e1}-\sigma_{e3}).$$

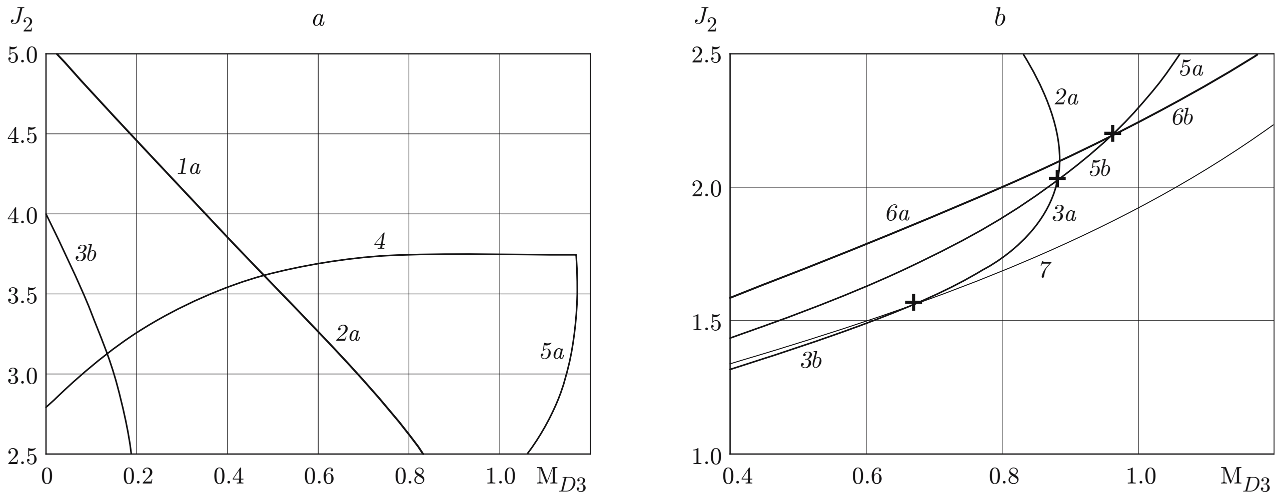

Fig. 3. Intensity of the second shock wave on the Mach number of the third shock wave in the neighborhood of the von Neumann configuration (a) and transitional triple configuration (b).

It follows from the latter relation that the dependences of M_{D2}, J_2, and other parameters on M_{D3} are continuous if the number of the solution changes as M_{D3} passes through zero. Figure 2 shows the numerical solutions of the problem (curve 3 corresponding to the condition $M_{D3} < 0$ continuously passes to curve 1 corresponding to the condition $M_{D3} > 0$).

Figure 3 shows the intensity of the second shock wave J_2 as a function of the Mach number of the third shock wave M_{D3} for $J_1 = 2$ for triple configurations of four types: TC-1 (curve 1), TC-2 (curve 2), and TC-3 (curve 3); there is no solution for TC-4; the curves for the solutions 1 and 2 are marked by the letters a and b, respectively.

3. Boundaries of Solution Existence. We study the boundaries of existence of solutions of all types with variation of the parameter M_{D3} for different values of J_1. The boundary separating the domains of existence of the solutions of the types TC-1 and TC-2 is the von Neumann configuration of shock waves in which the third shock wave propagates normal to the gas flow; the von Neumann configuration is shown as curve 4 in Fig. 3a.

The triple configuration of shock waves where the second shock wave is normal to the incoming flow is called the transitional triple configuration (TTC). This configuration is described by the equation $\sigma_{e2} = \pi/2$. It follows from Figs. 1b and 1c that TTC is the boundary between the solutions of the types TC-2 and TC-3. With allowance for Eqs. (7)–(10), the condition for TTC yields an equation for σ_{e3} with the quantities M_0, M_{D1}, J_1, and J_2 as parameters:

$$\psi_3(\chi_2 I_2 + M_1)\sin\sigma_{e1}\cos\sigma_{e3} - \sqrt{(J_1+\varepsilon)/(J_1(1+\varepsilon J_1))}\,\psi_3 M_{D1}\cos\beta_1\cos\sigma_{e3}$$

$$-\sqrt{(J_1+\varepsilon)/(J_1(1+\varepsilon J_1))}\,(M_{D1}\sin\beta_1 - M_0\cos(\sigma_{e1}-\beta_1))\sin\sigma_{e3}$$

$$-(\chi_2 I_2 + M_1)\cos\sigma_{e1}\sin\sigma_{e3} + \sqrt{(J_1+\varepsilon)/(J_1(1+\varepsilon J_1))}\,\chi_3 I_3\cos(\sigma_{e1}-\beta_1) = 0. \qquad (12)$$

The value of M_{D3} is determined from Eq. (3) with the use of Eq. (12); the solution of Eq. (11) yields the quantity J_2 as a function of M_0, M_{D1}, and J_1. The resultant solution is plotted in Fig. 3 (curves 5a and 5b).

As was noted above, the lower boundary of the domain of solutions of the type TC-2 is determined by von Neumann configuration (see Fig. 3a); curves 5a and 6a in Fig. 3 are the upper boundaries of this domain for different ranges of J_1; no solutions of the type TC-2 exist outside the above-mentioned boundaries. The upper boundary of the domain of solutions of the type TC-3 in the same ranges of J_1 is formed by curves 6b and 5b (Fig. 3b). The solution 1 reaches this boundary. The upper boundary of the domain of existence of the solution 2 of the type TC-3 is determined by the condition $\hat{D} = 0$; if this condition is satisfied, the solution 2 transforms to the solution 1. Part of this boundary is shown by curve 7 in Fig. 3b. Curves 6 restrict the domain of existence of solutions of the types TC-1, TC-2, and TC-3 from above.

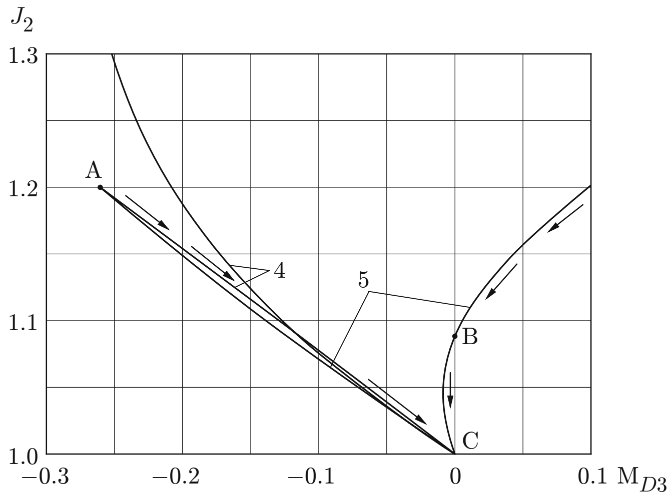

Fig. 4. Intensity of the second shock wave versus the Mach number of the third shock wave in the neighborhood of the weak second shock wave.

If the condition $\hat{D} = 0$ is satisfied, the solution of the problem reduces to system (9), (11) with respect to the unknowns J_2 and M_{D3}. There are three independent variables: M_0, M_{D1}, and J_1. As the basic unknown, we use the angle of inclination of the third shock wave σ_{e3}; hence, we have

$$J_2 = ((1+\varepsilon)(\mathrm{M}_{D3} - \mathrm{M}_0 \sin\sigma_{e3})^2 - \varepsilon)/J_1. \tag{13}$$

Relations (8)–(10) can be transformed to a quadratic equation with respect to M_{D3}

$$a\mathrm{M}_{D3}^2 + 2b\mathrm{M}_{D3} + c = 0, \tag{14}$$

where

$$a = 1 - ((1+\varepsilon J_1)/(J_1+\varepsilon))\sin^2(\psi_3\sigma_{e1} - \sigma_{e3}),$$

$$b = \sqrt{J_1(1+\varepsilon J_1)/(J_1+\varepsilon)}\,\mathrm{M}_1 \sin(\psi_3\sigma_{e1} - \sigma_{e3})\cos(\sigma_{e1} - \beta_1) - \psi_3\mathrm{M}_{D1}\cos(\psi_3\sigma_{e1} - \sigma_{e3}) + (1-a)\mathrm{M}_0\sin\sigma_{e3},$$

$$c = (1-a)(J_1\mathrm{M}_1^2 - \varepsilon(J_1-1)/(1+\varepsilon) - \mathrm{M}_0^2\sin^2\sigma_{e3})c$$

$$- 2\sqrt{J_1(1+\varepsilon J_1)/(J_1+\varepsilon)}\,\psi_3\mathrm{M}_{D1}\cos(\psi_3\sigma_{e3} - \beta_1)\mathrm{M}_1\sin(\psi_3\sigma_{e1} - \sigma_{e3}) + \mathrm{M}_{D1}^2.$$

Curve 7 in Fig. 3b obtained by solving Eq. (11) with the use of Eqs. (13) and (14) for TC-3 corresponds to the points of transition from the solution 1 to the solution 2 for different values of J_1.

We further consider the point of intersection of the TTC and von Neumann configuration curves, i.e., a point in the space $(J_1, J_2, \mathrm{M}_{D3})$ that is common for TTC and von Neumann configuration. To find this point, we substitute the von Neumann configuration condition $\sigma_{e3} = \pi/2$ into the TTC equation (12). After certain transformations, we obtain a quadratic equation with respect to I_2:

$$\Big(\frac{J_1+\varepsilon}{1+\varepsilon J_1}\cos^2(\sigma_{e1} - \beta_1) - \cos^2\sigma_{e1}\Big)I_2^2 - 2\chi_2 A\cos\sigma_{e1} I_2$$

$$- \Big(\frac{\varepsilon(J_1-1)(J_1+\varepsilon)}{(1+\varepsilon)(1+\varepsilon J_1)J_1}\cos^2(\sigma_{e1} - \beta_1) + A^2\Big) = 0,$$

$$A = \mathrm{M}_1\cos\sigma_{e1} + \sqrt{(J_1+\varepsilon)/(J_1(1+\varepsilon J_1))}\,(\mathrm{M}_{D1}\sin\beta_1 - \mathrm{M}_0\cos(\sigma_{e1} - \beta_1)).$$

Substituting the solution I_2 into Eq. (11), we obtain J_1, and then we find J_2. Figure 4 shows the von Neumann configuration curve (curve 4) in the neighborhood of the point of intersection of the curves that refer to TTC and von Neumann configuration (point A). At this point, we have $J_1 = 10.20$, $J_2 = 1.20$, and $\mathrm{M}_{D3} = -0.26$. TTC is presented by two segments of curve 5 converging at the point $\mathrm{M}_{D3} = 0$, $J_2 = 1$ (point C) corresponding to the weak second shock wave in the steady-state problem, for $J_1 = 10.22$ and 9.09. The segment of curve 5 between the points A and C does not have any other common points with curve 4 in the space $(J_1, J_2, \mathrm{M}_{D3})$. The second segment ($J_1 \le 9.09$) exists and is continuous for all $J_1 > 1$. The arrows on the TTC curve indicate the direction of the increase in the parameter J_1. The calculations show that no TTC exists in the interval $9.09 \le J_1 \le 10.20$. Figure 4 also contains the point B corresponding to TTC in the steady-state problem considered in [4] ($\sigma_{e1} = 1.09$).

We also consider the neighborhood of the point with the coordinates $\mathrm{M}_{D3} = 0$, $J_2 = 1$. At this point, we linearize Eqs. (12) and (11), with rejected terms of the order of M_{D3}^2, $(J_2 - 1)^2$, and higher. As a result, we obtain the following relation for J_1:

$$(1+\varepsilon J_1)(2J_1^2 + 3\varepsilon J_1^2 + 2J_1 + 4\varepsilon J_1 + 4\varepsilon^2 J_1 + \varepsilon) \pm (J_1 - 1)^2 \sqrt{\varepsilon(J_1+\varepsilon)(1+\varepsilon J_1)}$$

$$= -(1+\varepsilon) J_1 (3\varepsilon J_1^2 + 4J_1 + 2\varepsilon J_1 + 4\varepsilon^2 J_1 + 3\varepsilon)\chi_2 \mathrm{M}_1. \qquad (15)$$

Equation (5) yields the relation for M_1:

$$J_1(1+\varepsilon J_1)\mathrm{M}_1^2 = (J_1+\varepsilon)\mathrm{M}^2 - (1-\varepsilon)(J_1^2 - 1).$$

The solution of Eq. (15) corresponding to the plus sign yields the lower value $J_1 = 9.09$, while the solution corresponding to the minus sign yields the higher value $J_1 = 10.22$. These values of J_1 coincide with the values of J_1 obtained directly from Eqs. (12) and (11). The existence of two segments of the TTC curve is demonstrated analytically and numerically. These segments require additional studies to be performed.

Conclusions. Expressions relating the basic gas-dynamic parameters in the general case of formation of triple configurations by interfering traveling shock waves are derived. The calculations are performed in a coordinate system fitted to the first shock wave ($\mathrm{M}_{D1} = 0$), i.e., the interference of a quiescent shock wave and a traveling shock wave is studied. In particular, a flow of a diatomic gas ($\gamma = 7/5$) with a free-stream Mach number $\mathrm{M}_0 = 3$ is considered. It is demonstrated that solutions of three types (TC-1, TC-2, and TC-3) exist in the case considered, as in the case of interference of shock waves (steady-state problem). Domains of existence of these types of solutions are obtained, as well as the transitional triple configuration and the von Neumann configuration, which restrict these solutions.

It is found (for $J_1 = 2$) that the angle of inclination of the second shock wave with $\mathrm{M}_{D3} > 0$ is substantially greater than that in the steady-state case. For the values of the parameters considered, the intensities of the second and third shock waves (J_2 and J_3, respectively) drastically decrease for the solutions of the types TC-1 and TC-3. In the case of the transitional triple configuration, the intensity J_2 decreases from the value $J_2 = 3.2$ for $J_1 = 1$ to the value $J_2 = 1$ in the steady-state case ($J_1 = 9.09$).

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